

Compromised Peacebuilding

Online Appendix

In this appendix we provide detailed proofs for the two propositions in the main text. We first show that there is a unique efficient (i.e., no delay) stationary SPE to the two-player game. We then show that the equilibrium is the unique SPE to the game. A similar proof then is applied to the three-player game.

An efficient stationary SPE to the two-player game is a set of (x, m, y, n) that simultaneously solves the maximization problems for P and G described below. P 's optimization problem is:

$$\begin{aligned} \max_{x,m} \quad & u_P(x) - m \\ \text{s.t.} \quad & \frac{u_G(x) + m}{1 - \delta_G} \geq u_G(0) + \frac{\delta_G(u_G(y) + n)}{1 - \delta_G} \\ & M \geq m \geq 0 \end{aligned} \tag{1}$$

Equation (1) can be rewritten as $u_G(x) - u_G(0) + m \geq \delta_G[u_G(y) - u_G(0) + n]$. Since what matters for the inequality constraint is only the utility difference between a new policy and the status quo (i.e., the disagreement point), without loss of generality, let $u_G(0) = 0$. Additionally, let $A = \delta_G(u_G(y) + n)$, which is G 's continuation value. Then the problem can be restated as:

$$\begin{aligned} \max_{x,m} \quad & u_P(x) - m \\ \text{s.t.} \quad & u_G(x) + m \geq A \\ & M \geq m \geq 0 \end{aligned} \tag{2}$$

Similarly, the government's optimization problem is:

$$\begin{aligned} \max_{y,n} \quad & u_G(y) + n \\ \text{s.t.} \quad & \frac{u_P(y) - n}{1 - \delta_P} \geq u_P(0) + \frac{\delta_P(u_P(x) - m)}{1 - \delta_P} \\ & M \geq n \geq 0 \end{aligned}$$

Again, without loss of generality, let $u_P(0) = 0$. Also, let $B = \delta_P(u_P(x) - m)$, which is P 's continuation value. Then the problem can be restated as:

$$\begin{aligned} \max_{y,n} \quad & u_G(y) + n \\ \text{s.t.} \quad & u_P(y) - n \geq B \\ & M \geq n \geq 0 \end{aligned} \tag{3}$$

In what follows, we first derive the best response functions for P and G , $BR_P(A)$ and $BR_G(B)$, respectively. From the best response functions, we obtain the functions of the continuation values for P and G , $\delta_P F(A)$ and $\delta_G H(B)$. We then show that $F(A)$ and $H(B)$ are both continuous, decreasing, and concave. Finally, using the properties of $F(A)$ and $H(B)$, we prove the existence and uniqueness of the equilibrium in propositions 1 and 2.

Deriving the best response functions

First, observe that if inequality (2) does not bind, then $m = A - u_G(x)$ at the optimum, which reduces P 's optimization problem to $\max_x u_P(x) + u_G(x)$. Similarly, if (3) does not bind, then $n = u_P(y) - B$ at the optimum, and it reduces G 's optimization problem to $\max_y u_P(y) + u_G(y)$. This means when (2) and (3) do not bind, the optimal policies, x^* and y^* , are the solutions to $u'_P(x) = -u'_G(x)$, and $x^* = y^*$. Recall that g is G 's ideal point, and 1 is P 's ideal point; therefore, $u'_G(g) = 0$, $u'_P(1) = 0$, $u'_G(1) < 0$, and $u'_P(g) > 0$. It follows $x^* = y^* \in (g, 1)$.

Since A is a constant for P 's optimization problem, we can solve for (x, m) given any A . Define the following threshold values of A which will help derive P 's best response function. Let $A_0^{**} = \delta_G u_G(1)$, $A_0^* = u_G(x^*)$, $A_M^* = u_G(x^*) + M$, and $A_M^{**} = \delta_G(u_G(g) + M)$. These values are in ascending order because for G the best outcome in the next period is (g, M) , and the worst outcome is $(1, 0)$.¹ Moreover, since G cannot be forced to accept an offer that brings a worse payoff than the status quo, $A \in [\max\{0, A_0^{**}\}, A_M^{**}]$.² The best response function of P depends on G 's continuation value A , as well as whether satisfying G makes P worse off than being at the status quo. In particular, if G 's continuation value is A_M^{**} , then proposing a contract that is acceptable to G will mean that P receives at most $u_P(\bar{x}) - M$, where \bar{x} solves $u_G(\bar{x}) + M = \delta_G(u_G(g) + M)$; but if $u_P(\bar{x}) - M < 0$, then P is better off staying at the status quo.

We first consider the case $u_P(\bar{x}) - M \geq 0$. Then, P 's best response function given A is:

$$BR_P(A) = (x, m) = \begin{cases} (x : u_G(x) = A, m = 0) & \text{if } 0 \leq A < A_0^*; \\ (x^*, m = A - u_G(x^*)) & \text{if } A_0^* \leq A \leq A_M^*; \\ (x : u_G(x) = A - M, m = M) & \text{if } A_M^* < A \leq A_M^{**}. \end{cases}$$

The three cases correspond to whether equation (2) binds from below ($m = 0$), does not bind ($0 \leq m \leq M$), or binds from above ($m = M$) when P plays its best response. Because each pair of (x, m) from the best response function corresponds to a level of P 's utility, we can express P 's continuation value when G makes a proposal as follows: $\delta_P u_P(BR_P(A)) =$

¹Without loss of generality, we assume $A_M^* < A_M^{**}$, which can be satisfied by a sufficiently large δ_G .

²For the ease of exposition, we assume $A_0^{**} < 0$ in the derivation of P 's best response below. We discuss the case for $A_0^{**} > 0$ later in the proof for Proposition 1.

$\delta_P F(A)$, where

$$F(A) = \begin{cases} u_P(u_G^{-1}(A)) & \text{if } 0 \leq A < A_0^*; \\ u_P(x^*) + u_G(x^*) - A & \text{if } A_0^* \leq A \leq A_M^*; \\ u_P(u_G^{-1}(A - M)) - M & \text{if } A_M^* < A \leq A_M^{**}. \end{cases}$$

To simplify the notation, let $f(A) = u_P(u_G^{-1}(A))$. Then,

$$F(A) = \begin{cases} f(A) & \text{if } 0 \leq A < A_0^*; \\ f(A_0^*) + A_0^* - A & \text{if } A_0^* \leq A \leq A_M^*; \\ f(A - M) - M & \text{if } A_M^* < A \leq A_M^{**}. \end{cases}$$

If $u_P(\bar{x}) - M < 0$, on the other hand, then $F(A)$ needs to be slightly modified in order to take into account the possibility that for some high values of A , P is better off being at the status quo than satisfying G 's demand. Define an additional threshold $A_0 \in (A_M^*, A_M^{**})$, which is the level of G 's continuation value that makes P indifferent between offering a contract acceptable to G and being at the status quo.³ In other words, $A_0 = u_G(x_0) + M$, where $g < \bar{x} < x_0 < x^*$, such that $F(A_0) = 0$. Then, P 's continuation value when G makes a proposal is $\delta_P \bar{F}(A)$, where:

$$\bar{F}(A) = \begin{cases} f(A) & \text{if } 0 \leq A < A_0^*; \\ f(A_0^*) + A_0^* - A & \text{if } A_0^* \leq A \leq A_M^*; \\ f(A - M) - M & \text{if } A_M^* < A \leq A_0; \\ 0 & \text{if } A_0 < A \leq A_M^{**}. \end{cases}$$

Indeed, in the rest of the proof we assume that $u_P(\bar{x}) - M < 0$, because in a peacebuilding context it is highly unlikely that P is willing to pay the maximum resources to G only to revise the status quo to an outcome that is most preferable from G 's perspective. Note that the difference between $F(A)$ and $\bar{F}(A)$ is simply that the last case in $F(A)$ is split into two in $\bar{F}(A)$ because of an additional threshold, A_0 .

We now derive G 's best response function in a similar fashion as that of P 's. Let $B_0^{**} = \delta_P u_P(1)$, $B_0^* = u_P(x^*)$, $B_M^* = u_P(x^*) - M$, and $B_M^{**} = \delta_P(u_P(g) - M)$. These values are in descending order because for P in the next period the best outcome is $(1,0)$, and the worst outcome is (g, M) .⁴ Moreover, we know $B_M^{**} < 0$, and since P cannot be forced to accept an offer that brings a worse payoff than the status quo, $B \in [0, B_0^{**}]$. G 's best response function depends on P 's continuation value B , and whether G is better off being at the status quo or making the most favorable proposal from P 's perspective. We analyze both cases because

³The proof does not depend on where A_0 locates; assuming $A_0 \in (A_M^*, A_M^{**})$ allows us to demonstrate all possible cases of P 's best response function.

⁴Without loss of generality, we assume $B_M^* > B_M^{**}$, which can be satisfied by a sufficiently large δ_P .

both scenarios are empirically plausible. To do so, define \hat{x} such that $u_P(\hat{x}) = \delta_P u_P(1)$. It follows that $\hat{x} < 1$, and $u_G(1) < u_G(\hat{x})$. If P 's continuation value is B_0^{**} , i.e., P gets its most preferred outcome in the next period, then proposing a contract that is acceptable to P will mean that G at most receives $u_G(\hat{x})$.

Suppose $u_G(\hat{x}) \geq 0$, then G is at least as well off at \hat{x} as at the status quo. G 's best response function in this case is:

$$BR_G(B) = (y, n) = \begin{cases} (y : u_P(y) = B + M, n = M) & \text{if } 0 \leq B \leq B_M^*; \\ (x^*, n = u_P(x^*) - B) & \text{if } B_M^* \leq B \leq B_0^*; \\ (y : u_P(y) = B, n = 0) & \text{if } B_0^* \leq B \leq B_0^{**}. \end{cases}$$

Let $h(B) = u_G(u_P^{-1}(B))$. Then G 's continuation value when P makes a proposal is $\delta_G u_G(BR_G(B)) = \delta_G H(B)$, where

$$H(B) = \begin{cases} h(B + M) + M & \text{if } 0 \leq B < B_M^*; \\ h(B_0^*) + B_0^* - B & \text{if } B_M^* \leq B \leq B_0^*; \\ h(B) & \text{if } B_0^* < B \leq B_0^{**}. \end{cases}$$

Now suppose $u_G(\hat{x}) < 0$ (which implies $A_0^{**} = \delta_G u_G(1) < 0$). Then for some high values of B , G is better off being at the status quo than making a proposal acceptable to P . Let $B_0 \in (B_0^*, B_0^{**})$ be the level of P 's continuation value that makes G indifferent between proposing a contract acceptable to P and being at the status quo.⁵ In other words, $B_0 = u_P(y_0)$, where $x^* < y_0 < \hat{x} < 1$, such that $H(B_0) = 0$. Then, G 's continuation value when P makes a proposal is $\delta_G \bar{H}(B)$, where:

$$\bar{H}(B) = \begin{cases} h(B + M) + M & \text{if } 0 \leq B < B_M^*; \\ h(B_0^*) + B_0^* - B & \text{if } B_M^* \leq B \leq B_0^*; \\ h(B) & \text{if } B_0^* < B \leq B_0; \\ 0 & \text{if } B_0 < B \leq B_0^{**}. \end{cases}$$

To summarize, the function of P 's continuation values is $\delta_P \bar{F}(A)$; the function of G 's continuation values is $\delta_G H(B)$ if $u_G(\hat{x}) \geq 0$, and $\delta_G \bar{H}(B)$ if $u_G(\hat{x}) < 0$.

Show that $F(A)$ and $H(B)$ are both continuous, decreasing, and concave

To prove the uniqueness of the equilibrium in the propositions, we want to show that there is a unique fixed point for $\delta_P \bar{F}(A)$ and $\delta_G H(B)$, and for $\delta_P \bar{F}(A)$ and $\delta_G \bar{H}(B)$. We therefore need to understand the properties of $F(A)$ and $H(B)$. Lemma 1 shows that $F(A)$ is a continuous decreasing concave function.⁶ By an analogous proof, we can show that $H(B)$ shares the same properties.

⁵The condition is equivalent to $u_G(1) < 0 < u_G(x^*)$. Again, a different location of B_0 does not change the result.

⁶We prove for the case $A_0^{**} \leq 0$. The case for $A_0^{**} > 0$ is similar.

Lemma 1. $F(A)$ is continuous, decreasing, and concave.

Proof. We discuss the three intervals of $F(A)$ in turn.

1. If $0 \leq A < A_0^*$, $f'(A) = u'_P(u_G^{-1}(A))/u'_G(u_G^{-1}(A))$. Note that $u'_P > 0$ and is decreasing, $u'_G < 0$ and is decreasing, and u_G^{-1} is decreasing. Therefore, $f'(A) < 0$, and moreover, $|f'(A)| = u'_P(u_G^{-1}(A))/|u'_G(u_G^{-1}(A))|$ is increasing. Thus $f(A)$ is a decreasing concave function. Also note that $f'(A_0^*) = \frac{u'_P(x^*)}{u'_G(x^*)} = -1$.
2. If $A_0^* \leq A \leq A_M^*$, $F(A)$ is linear with a slope of -1 .
3. If $A_M^* < A \leq A_M^{**}$, $f'(A-M) = u'_P(u_G^{-1}(A-M))/u'_G(u_G^{-1}(A-M))$. As in case 1 we find that $f(A-M)$ is a decreasing concave function. Moreover, $f'(A_M^* - M) = \frac{u'_P(x^*)}{u'_G(x^*)} = -1$.

We conclude that $F(A)$ is continuous, decreasing, and concave in each of the three intervals, and moreover, $F(A)$ connects smoothly at the thresholds of the intervals, and so $F(A)$ has the stated properties over its entire feasible range. \square

Proposition 1 *There is a unique subgame perfect equilibrium to the game.*

- (1) *If $u_G(1) < 0$, then the equilibrium produces a compromised policy outcome;*
- (2) *If $u_G(1) \geq 0$, then the equilibrium policy outcome is compromised if $\delta_G > \frac{u_G(1)}{u_G(\hat{x})}$ and cooperative if $\delta_G \leq \frac{u_G(1)}{u_G(\hat{x})}$, where \hat{x} is defined by $u_P(\hat{x}) = \delta_P u_P(1)$. In the cooperative equilibrium, the policy outcome is the peacebuilders' ideal point, and there is no resource transfer from the peacebuilders to the government.*

Proof. We first show that there is a unique efficient stationary SPE by proving that there is a unique fixed point for the best response functions that we have derived. We then show that it is the unique SPE to the game.

(a) Suppose $u_G(\hat{x}) < 0$, and thus $u_G(1) < 0$. Then the system of equations to solve for the fixed point is:

$$\begin{cases} B = \delta_P \bar{F}(A) \\ A = \delta_G \bar{H}(B). \end{cases}$$

where $A \in [0, A_M^{**}]$ and $B \in [0, B_0^{**}]$. Suppose we draw diagrams of $\delta_P \bar{F}(A)$ and $\delta_G \bar{H}(B)$, using A as the horizontal axis and B as the vertical axis. Then the feasible ranges are defined by $[0, A_M^{**}]$ on the horizontal axis, and $[0, B_0^{**}]$ on the vertical axis. Note that $\bar{F}(A)$ is just $F(A)$ truncated at $B = 0$, and $\bar{H}(B)$ is just $H(B)$ truncated at $A = 0$; therefore, $\bar{F}(A)$ and $\bar{H}(B)$ are also continuous, decreasing, and concave in $[0, A_0]$ and $[0, B_0]$, respectively. Therefore, a unique fixed point exists if the intercepts of the two functions on the horizontal and vertical axes satisfy the following conditions:

Condition 1: $\delta_G H(0) < A_0$ where A_0 solves $B(A_0) = 0$;

Condition 2: $\delta_P F(0) < B_0$ where B_0 solves $A(B_0) = 0$.

For condition 1, since $B(A_0) = \delta_P F(A_0) = 0$, $A_0 = F^{-1}(0) = H(0)$. Hence, $\delta_G H(0) < A_0$. For condition 2, since $A(B_0) = \delta_G H(B_0) = 0$, $B_0 = H^{-1}(0) = F(0)$. Hence, $\delta_P F(0) < B_0$. Therefore, there is always a unique fixed point for this case.

(b) Suppose $u_G(\hat{x}) \geq 0$. We need to consider two cases. If it is still the case that $A_0^{**} = \delta_G u_G(1) \leq 0$, then $A \in [0, A_M^{**}]$, and $B \in [0, B_0^{**}]$. This is a similar case to (a), and the same proof can be applied to show that there is a unique fixed point.

Now suppose $u_G(\hat{x}) \geq 0$, and also $A_0^{**} = \delta_G u_G(1) > 0$. Then, $A \in [A_0^{**}, A_M^{**}]$, while $B \in [0, B_0^{**}]$. In this case, because $u_G(1) > 0$, P 's best response for all $A \in [A_0^{**}, u_G(1)]$ is to offer $(1, 0)$ and it will be accepted by G . That is, when A changes from $\delta_G u_G(1)$ to $u_G(1)$, P can always propose $(1, 0)$ and the government will accept. Consequently, $\bar{F}(A) = u_P(1)$ for $A \in [A_0^{**}, u_G(1)]$. For $A \in (u_G(1), A_M^{**}]$, $\bar{F}(A)$ has the same shape as that for $A_0^{**} \leq 0$. To summarize, for this case:

$$\bar{F}(A) = \begin{cases} u_P(1) & \text{if } A_0^{**} \leq A \leq u_G(1); \\ f(A) & \text{if } u_G(1) < A < A_0^*; \\ f(A_0^*) + A_0^* - A & \text{if } A_0^* \leq A \leq A_M^*; \\ f(A - M) - M & \text{if } A_M^* < A \leq A_0; \\ 0 & \text{if } A_0 < A \leq A_M^{**}. \end{cases}$$

Then the system of equations to solve for the fixed point is:

$$\begin{cases} B = \delta_P \bar{F}(A) \\ A = \delta_G H(B). \end{cases}$$

Using a proof that is similar to case (a), it can be shown that there is always a fixed point. Moreover, if $\delta_G u_G(\hat{x}) > u_G(1)$, or $\delta_G > \frac{u_G(1)}{u_G(\hat{x})}$, then there is a compromised outcome; if $\delta_G \leq \frac{u_G(1)}{u_G(\hat{x})}$, then there is a cooperative (or uncompromised) outcome. To interpret the condition, note that $u_P(\hat{x}) = \delta_P u_P(1)$, so \hat{x} is an increasing function of δ_P ; therefore $u_G(\hat{x})$ is a decreasing function of δ_P . This means that the more patient P is, the less likely it is that we will see a compromised outcome. Combining these results, we now found the SPE characterized in Proposition 1.

For the uniqueness of the SPE, we invoke Theorem 3.2 in Muthoo (1999, pp.60-62). Let Ω denote the set of all possible utility pairs for a generalized Rubinstein model such as ours. Furthermore, let Ω^e denote the Pareto frontier of Ω . Suppose Ω^e is the graph of a concave function, ϕ , whose domain is an interval $I_A \subseteq \Re$ and range an interval $I_B \subseteq \Re$, with $0 \in I_A$, $0 \in I_B$ and $\phi(0) > 0$.⁷ Theorem 3.2 says if there is a unique no delay stationary SPE

⁷See assumption 3.1, Muthoo p.60

to such a game, then it is also the unique SPE to the game. The result can be directly applied to our model, where $\Omega^e = F(A)$ satisfies all the conditions required for the theorem to hold. In other words, the unique efficient stationary SPE we found *is* the unique SPE to the game.⁸ \square

Proposition 2 *There is a unique subgame perfect equilibrium to the three-player game and the equilibrium policy outcome is a compromised one. Specifically, the policy outcome is \tilde{x} if $\tilde{x} \leq g$, and $x \in (g, \tilde{x}]$ if $\tilde{x} > g$.*

Proof. Suppose there is a secondary elite, E , who is a veto player with an ideal point, e . Let $u_E(x)$ be E 's utility function that is continuous and strictly concave, and $l_E \geq 0$ be the total benefit for E from being recognized in a peace contract. E will accept a contract if $u_E(x) + l_E \geq u_E(0)$. Let $\tilde{x} \in (0, 1)$ solves the equality,⁹ i.e., if $x \leq \tilde{x}$, then E will accept the contract negotiated between P and G .

Consider the best response functions for P and G given the additional constraint imposed by \tilde{x} . There are three possibilities for \tilde{x} 's location: $\tilde{x} \leq g$, $g < \tilde{x} \leq x^*$, and $x^* < \tilde{x}$. Below we derive the best response functions for all the cases and show that there is a unique subgame perfect equilibrium.

Let $\tilde{A}_0 = u_G(\tilde{x})$, and $\tilde{A}_M = u_G(\tilde{x}) + M$. Similarly, let $\tilde{B}_0 = u_P(\tilde{x})$, and $\tilde{B}_M = u_P(\tilde{x}) - M$. Assume that if there is no acceptable contract to the other two players, then P and G will just choose the status quo as their best response.¹⁰ We maintain the assumption that $u_P(\bar{x}) - M < 0$.

Case 1: $\tilde{x} \leq g$. Then $u_P(\tilde{x}) - M < u_P(g) - M < 0$, so P is better off at the status quo than moving to \tilde{x} and transferring the maximum resources to G . Recall that $A_0 \in (A_M^*, A_M^{**})$ denotes G 's continuation value that makes P indifferent between staying at the status quo and making an offer acceptable to G . That is, $B(A_0) = 0$. Then $A_0 \in (\tilde{A}_0, \tilde{A}_M)$. Then, P 's best response function is:

$$BR_P(A) = \begin{cases} (\tilde{x}, m = 0) & \text{if } \max\{0, A_0^{**}\} \leq A < \tilde{A}_0; \\ (\tilde{x}, m = A - u_G(\tilde{x})) & \text{if } \tilde{A}_0 \leq A \leq A_0; \\ (0, 0) & \text{if } A_0 < A \leq A_M^{**}. \end{cases}$$

⁸There is a small technical difference between assumption 3.1 and our model for the case $u_G(1) > 0$, because the Pareto frontier in this case does not intersect with the two axes, thus the coordinate $(0,0)$ is not in the feasible set as required by the assumption. However, the purpose of the assumption is to ensure the existence of a stationary equilibrium, and since there is indeed a stationary equilibrium to the game, the logic of Theorem 3.2 applies.

⁹If $\tilde{x} \geq 1$, then the elite does not pose any constraint on the bargaining.

¹⁰In general, P and G can offer anything in such cases because the offer will be rejected by one of the other two players, and the resulting payoff is 0.

As in the two-player game, we can express P 's continuation values as a function of A : $\delta_P u_P(BR_P(A)) = \delta_P F(A)$, where

$$F(A) = \begin{cases} u_P(\tilde{x}) & \text{if } \max\{0, A_0^{**}\} \leq A < \tilde{A}_0; \\ u_P(\tilde{x}) + u_G(\tilde{x}) - A & \text{if } \tilde{A}_0 \leq A \leq A_0; \\ 0 & \text{if } A_0 < A \leq A_M^{**}. \end{cases}$$

For G , recall that $B_0 \in (B_0^*, B_0^{**})$ is P 's continuation value that makes G indifferent between staying at the status quo and making a proposal acceptable to P . Because $\tilde{B}_0 < B_0^* < B_0$, G 's best response function is:

$$BR_G(B) = \begin{cases} (\tilde{x}, n = u_P(\tilde{x}) - B) & \text{if } 0 \leq B \leq \tilde{B}_0; \\ (0, 0) & \text{if } \tilde{B}_0 < B \leq B_0^{**}. \end{cases}$$

Accordingly, the function of G 's continuation values is $\delta_G u_G(BR_P(B)) = \delta_G H(B)$, where

$$H(B) = \begin{cases} u_G(\tilde{x}) + u_P(\tilde{x}) - B & \text{if } 0 \leq B \leq \tilde{B}_0; \\ 0 & \text{if } \tilde{B}_0 < B \leq B_0^{**}. \end{cases}$$

By a proof analogous to that for Proposition 1 case (a), we can show that there is a unique fixed point in the set $[\max\{0, A_0^{**}\}, A_M^{**}] \times [0, B_0^{**}]$. Moreover, from the best response functions we can see that the equilibrium is (\tilde{x}, m) , where $0 \leq m < M$.

Case 2: $g < \tilde{x} \leq x^*$. In this case, because \tilde{x} is not very constraining, assume that $u_P(\tilde{x}) - M > 0$. That is, P prefers to pay the maximum resources to achieve \tilde{x} to staying at the status quo. Then $A_0 \in (\tilde{A}_M, A_M^{**})$.¹¹ Then, P 's best response function is as follows:

$$BR_P(A) = \begin{cases} (\tilde{x}, m = 0) & \text{if } \max\{0, A_0^{**}\} \leq A < \tilde{A}_0; \\ (\tilde{x}, m = A - u_G(\tilde{x})) & \text{if } \tilde{A}_0 \leq A \leq \tilde{A}_M; \\ (x : u_G(x) = A - M, m = M), \text{ where } g < x < \tilde{x}, & \text{if } \tilde{A}_M < A \leq A_0; \\ (0, 0) & \text{if } A_0 < A \leq A_M^{**}. \end{cases}$$

Accordingly, the continuation value of P is $\delta_P u_P(BR_P(A)) = \delta_P F(A)$, where

$$F(A) = \begin{cases} u_P(\tilde{x}) & \text{if } \max\{0, A_0^{**}\} \leq A < \tilde{A}_0; \\ u_P(\tilde{x}) + u_G(\tilde{x}) - A & \text{if } \tilde{A}_0 \leq A \leq \tilde{A}_M; \\ u_P(u_G^{-1}(A - M)) - M & \text{if } \tilde{A}_M < A \leq A_0; \\ 0 & \text{if } A_0 < A \leq A_M^{**}. \end{cases}$$

For G , again, since $\tilde{B}_0 < B_0$, G 's best response function is:

$$BR_G(B) = \begin{cases} (y : u_P(y) = B + M, n = M), \text{ where } g < y < \tilde{x}, & \text{if } 0 \leq B < \tilde{B}_M; \\ (\tilde{x}, n = u_P(\tilde{x}) - B) & \text{if } \tilde{B}_M \leq B \leq \tilde{B}_0; \\ (0, 0) & \text{if } \tilde{B}_0 < B \leq B_0^{**}. \end{cases}$$

¹¹As in the two-player game, the location of A_0 does not change the result.

Accordingly, the continuation value of G is $\delta_G u_G(BR_P(B)) = \delta_G H(B)$, where

$$H(B) = \begin{cases} u_G(u_P^{-1}(B+M)) + M & \text{if } 0 \leq B < \tilde{B}_M; \\ u_G(\tilde{x}) + u_P(\tilde{x}) - B & \text{if } \tilde{B}_M \leq B \leq \tilde{B}_0; \\ 0 & \text{if } \tilde{B}_0 < B \leq B_0^{**}. \end{cases}$$

By a proof analogous to that for Proposition 1 case (a), we can show that there is a unique fixed point in the set $[\max\{0, A_0^{**}\}, A_M^{**}] \times [0, B_0^{**}]$. Moreover, from the best response functions we can see that the equilibrium outcome is either (\tilde{x}, m) where $0 \leq m < M$, or (x, M) where $x \in (g, \tilde{x})$.

Case 3: $\tilde{x} > x^*$. This means $u_P(\tilde{x}) - M > u_P(x^*) - M > 0$. So we have $A_0^{**} < \tilde{A}_0 < A_0^* < \tilde{A}_M < A_M^* < A_0 < A_M^{**}$. Because \tilde{x} is sufficiently high in this case, the best response function in this case is very similar to the two-player case:

$$BR_P(A) = \begin{cases} (\tilde{x}, m = 0) & \text{if } \max\{0, A_0^{**}\} \leq A < \tilde{A}_0; \\ (x : u_G(x) = A, m = 0) & \text{if } \tilde{A}_0 \leq A \leq A_0^*; \\ (x^*, m = A - u_G(x^*)) & \text{if } A_0^* < A \leq A_M^*; \\ (x : u_G(x) = A - M, m = M) & \text{if } A_M^* < A < A_0; \\ (0, 0) & \text{if } A_0 \leq A \leq A_M^{**} \end{cases}$$

G 's best response function is:

$$BR_G(B) = (y, n) = \begin{cases} (y : u_P(y) = B + M, n = M) & \text{if } 0 \leq B < B_M^*; \\ (x^*, n = u_P(x^*) - B) & \text{if } B_M^* \leq B \leq B_0^*; \\ (y : u_P(y) = B, n = 0) & \text{if } B_0^* < B \leq \min\{\tilde{B}_0, B_0\}; \\ (0, 0) & \text{if } \min\{\tilde{B}_0, B_0\} < B \leq B_0^{**}. \end{cases}$$

We can then derive $\delta_P F(A)$ and $\delta_G H(B)$ in the same fashion as that for all other cases. Furthermore, by a proof analogous to that for Proposition 1 case (a), we can show that there is a unique fixed point in the set $[\max\{0, A_0^{**}\}, A_M^{**}] \times [0, B_0^{**}]$. Moreover, from the best response functions we can see that the equilibrium outcome is one of the following: $(x, 0)$ where $x^* < x \leq \tilde{x}$, (x^*, m) where $0 < m < M$, and (x, M) where $x \in (g, x^*)$.

We have shown thus far that there always exists a unique stationary SPE to the three-player game. The existence of the veto player puts a limit on what P can realistically expect if it is to reach an agreement accepted by the other two actors. Consequently P proposes a contract that is often more modest than that in the two-player game. Finally, It can be shown that the equilibrium is also the unique SPE to the game by the same theorem (Muthoo 1999, pp.60-62) invoked earlier in the proof of Proposition 1. \square